# *Electrostatics in Vacuum*

(CO-5 PART-II)

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# Concept of Electric field

The electric field at a point due to source charge is defined as the ***electrostatic force*** per unit positive charge acting on a small test charge placed at that point without disturbing the source charge. It is a ***vector quantity***, whose direction in which positive test charge tends to move. Its unit is ***Newton/coulomb***. If $q_0$ is a test charge experience a force F then electric field at that point is given by

$$E = F/\ q_0$$

or $$E = Q/4\pi\varepsilon_0 r^2$$ where $Q = \Sigma q =$ total charge present on the surface.

Electric field due to n number of charges distributed on the surface is such that $q_1, q_2, q_3 \ldots\ldots q_n$ are placed at a distance $r_1, r_2, r_3 \ldots\ldots r_n$ from the test charge then electric field is given by

$$E=(q_1+q_2+q_3+\ldots\ldots\ldots q_n)\ /\ 4\pi\varepsilon 0(r_1{}^2+r_2{}^2+r_3{}^2+\ldots\ldots\ldots\ldots r_n{}^2)$$

$$E = 1/4\pi\varepsilon_0\ \Sigma(q/r^2\ )$$

# Concept of Electrostatic Potential

- Generally, electric potential tells the ***direction of flow*** of positive charge.
- Flow of charge is possible between two points only when there is a ***potential difference occur*** between them.
- The direction of charge flow does not depend on the ***total amount of charge*** present in the conductor.
- Positive charges always move from ***high potential to low potential.***
- The potential of earth is considered ***as zero***.
- If a ***charge given*** to a conductor, its potential ***increases.***
- Potential of conductor is ***inversely*** proportional to the ***radius***.
- On the ***presence of uncharged conductor*** near to conductor, then potential is decreases.

# Concept of Electrostatic Potential

The electrostatic potential at a point a in the field is equal to the ***amount of work done*** in moving a unit positive test charge from infinity to that point against the electrostatic force, along any path.

$$V_a = W_{\infty \rightarrow a} / q_0$$

electrostatic potential is a ***scalar quantity***. Is unit is ***Joule/Coulomb or Volt.***

$$V = Q/4\pi\varepsilon_0 r$$

Potential between two points a and b is given by

$$V = -\int_a^b E \cdot dl$$

Potential due to n number of charges distributed on the surface is such that $q_1, q_2, q_3 \ldots\ldots q_n$ are placed at a distance $r_1, r_2, r_3 \ldots\ldots r_n$ from the test charge then electric potential is given by

$$V = (q_1 + q_2 + q_3 + \ldots\ldots\ldots q_n) / 4\pi\varepsilon 0 (r_1 + r_2 + r_3 + \ldots\ldots\ldots\ldots r_n)$$

$$V = 1/4\pi\varepsilon_0 \, \Sigma(q/r)$$

# Maxwell's First equation

Maxwell's first equation is based on the *gauss law of electrostatic*. According to this law electric flux passing through any closed surface is $1/\varepsilon_0$ of the total charge present inside the surface.

$$\phi_e = \text{Total charge} / \varepsilon_0$$

$$\phi_e = \Sigma q / \varepsilon_0$$

Where $\Sigma q$ =total charge present on the surface

$\phi_e = \iint \vec{E}.\vec{ds}$ =electric flux passing through the electric field,

$$\iint \vec{E}.\vec{ds} = \Sigma q / \varepsilon_0$$

or $$\iint \vec{E}.\vec{ds} = \iiint \rho dv / \varepsilon_0$$

or $$\varepsilon_0 \iint \vec{E}.\vec{ds} = \iiint \rho dv$$

This is *integral form* of Maxwell's first equation.

Contd ……

$$\varepsilon_0 \iint \vec{E}.\vec{ds} = \iiint \rho dv \quad .......(1)$$

By using gauss divergence theorem

$$\iint \vec{E}.\vec{ds} = \iiint \text{div}\, \vec{E}\, dv$$

Equation (1) become as

$$\varepsilon_0 \iiint \text{div}\, \vec{E}\, dv = \iiint \rho dv$$

$$\iiint (\text{ div}\, \vec{E} - \rho/\varepsilon_0 )dv = 0$$

$$\text{div}\, \vec{E} - \rho/\ \varepsilon_0 = 0$$

$$\text{div}\, \vec{E} = \rho/\varepsilon_0$$

This is the Maxwell's first equation in *differential form*.

$$\varepsilon_0\ \text{div}\, \vec{E} = \rho$$

$$\text{div}\, ( \varepsilon_0 \vec{E} ) = \rho \qquad \text{since } \vec{D} = \varepsilon_0 \vec{E}$$

$\mu_0 H$

$$\text{div}\, \vec{D} = \rho$$

This is another form of Maxwell's first equation..

# Maxwell's Second equation

Maxwell's second equation express the ***gauss law of magnetostatic***. According to this law magnetic flux passing through any closed magnetic field is always zero.

$$\phi_B = \iint \vec{B}.\vec{ds} = 0$$

This is ***integral form*** of Maxwell's second equation. By using gauss divergence theorem

$$\iint \vec{B}.\vec{ds} = \iiint \mathrm{div}\,\vec{B}\,dv.$$

Therefore

$$\iiint \mathrm{div}\,\vec{B}\,dv = 0$$

or

$$\mathrm{div}\,\vec{B} = 0$$

since $B = \mu_0 H$

or

$$\mathrm{div}\,\vec{H} = 0$$

This is the Maxwell's second equation in ***differential form***.

# Maxwell's Third equation

*Maxwell's third equation based on Faraday's law of electromagnetic induction. According to this law whenever magnetic flux associated with a circuit change, induced e.m.f. is produced. The magnitude of induced e.m.f. is directly proportional to the rate of change of magnetic flux linked with the circuit. Mathematically,*

$$e.m.f. = -\frac{\partial \phi_B}{\partial t}$$

where $\phi_B$ magnetic flux linked with circuit.

$$\oint \vec{E} \cdot dl = -\frac{\partial}{\partial t}\phi_B$$

but

$$\phi_B = \iint_s \vec{B} \cdot \vec{ds}$$

or

$$\oint \vec{E} \cdot dl = -\iint_s \left(\frac{\partial \vec{B}}{\partial t}\right) \cdot \vec{ds}$$

or

$$\oint \vec{E} \cdot dl = -\iint_s \left(\frac{\partial \vec{B}}{\partial t}\right) \cdot \vec{ds}$$

... well's first

This is *integral form* of Maxwell's third equation.

$$\oint \vec{E}\cdot\vec{dl} = \iint_s \left(\frac{\partial \vec{B}}{\partial t}\right)\cdot\vec{ds} \quad \ldots\ldots (1)$$

by using stock's theorem.

$$\oint \vec{E}\cdot\vec{dl} = \iint_s \text{curl}\,\vec{E}\cdot\vec{ds}$$

equation (1) become as

$$\iint_s \text{curl}\,\vec{E}\cdot\vec{ds} = -\iint_s \left(\frac{\partial \vec{B}}{\partial t}\right)\cdot\vec{ds}$$

or

$$\iint_s \left(\text{curl}\,\vec{E} + \frac{\partial \vec{B}}{\partial t}\right)\cdot\vec{ds} = 0$$

therfere $\quad \text{curl}\,\vec{E} + \dfrac{\partial \vec{B}}{\partial t} = 0$

or $\quad \text{curl}\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

or $\quad \vec{\nabla}\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

But $\quad \vec{B} = \mu_0 \vec{H} \quad \& \quad \vec{D} = \epsilon_0 \vec{E}$

or $\quad \text{curl}\,\vec{E} = -\mu_0 \dfrac{\partial \vec{H}}{\partial t}$

or $\quad \text{curl}\,\vec{D} = -\epsilon_0 \mu_0 \dfrac{\partial \vec{H}}{\partial t}$



This is *Differential form* of Maxwell's third equation.

# Maxwell's Fourth equation

*Maxwell's fourth equation derived from* ***Ampere's current law.*** *According to this law the line integral of magnetic field over a closed path is equal to $\mu_0$ times of the net current flowing through the closed path. Mathematically,*

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

but $I = I_c + I_d$

therfore

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 [I_c + I_d]$$

In term of current density

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left[\oint J \cdot ds + \oint \vec{I_d} \cdot \vec{ds}\right]$$

where $I_c$ = current in the circuit

$I_d$ = displacement current

This is ***integral form*** of Maxwell's fourth equation.

by using stock's theorem

$$\oint B \cdot dl = \iint_S \text{curl} \vec{B} \cdot \vec{ds}$$

then from eq$^n$ (1)

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \iint_S \left( J + \frac{\partial D}{\partial t} \right) \cdot ds$$

$$\iint_s \text{curl} \vec{B} \, ds = \mu_0 \iint_S \left( J + \frac{\partial D}{\partial t} \right) ds$$

$$\therefore \text{curl} \vec{B} = \mu_0 \left[ J + \frac{\partial D}{\partial t} \right]$$

but $B = \mu_0 H$ & $D = \epsilon_0 E$

$$\therefore \text{curl} H = J + \epsilon_0 \frac{\partial E}{\partial t}$$

This is ***Differential form*** of Maxwell's fourth equation.

# MAXWELL's EQUATION

| EQUATION | Diffrential form | Integral form |
| --- | --- | --- |
| Maxwell's First equation | $div \vec{E} = \rho / \epsilon_0$ or $div \vec{D} = \rho$ | $\iint \vec{E} \cdot \vec{ds} = \frac{\rho}{\epsilon_0} \iiint \rho dv$ or $\iint \vec{D} \cdot \vec{ds} = \iiint \rho dv$ |
| Maxwell's Second equation | $div \vec{B} = 0$ or $div \vec{H} = 0$ | $\iint \vec{B} \cdot \vec{ds} = 0$ or $\iint \vec{H} \cdot \vec{ds} = 0$ |
| Maxwell's Third equation | $curl \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or $curl \vec{E} = -\mu_0 \frac{\partial H}{\partial t}$ | $\iint_S \vec{E} \cdot dl = -\iint_S \frac{\partial \vec{B}}{\partial t} \vec{ds}$ or $\iint \vec{E} \, dl = -\mu_0 \iint_S \frac{\partial H}{\partial t} ds$ |
| Maxwell's Forth equation | $curl \vec{B} = \mu_0 \left[ J + \frac{\partial \vec{D}}{\partial t} \right]$ or $curl \vec{H} = J + \frac{\partial \vec{D}}{\partial t}$ | $\oint_C \vec{B} \cdot dl = \mu_0 \iint \left( J + \frac{\partial D}{\partial t} \right) ds$ or $\oint_C \vec{H} \cdot dl = \left( J + \frac{\partial \vec{D}}{\partial t} \right)$ |

# Significance of Maxwell's equations

***Significance of First equation -***It represent the ***Gauss's law in electrostatic*** for the static charges, it can be derived from coulomb's law.

***Significance of Second equation -***It express the ***Gauss's law in magnetostatic*** which signifies that magnetic lines of force formed closed curve and magnetic monopole does not exist.

***Significance of Third equation –***This is derived from ***Faraday's law of electromagnetic induction***. It signifies that an electric field is produced by changing magnetic field.

***Significance of Fourth equation –***It is generalized form of ***Ampere's law***. It signifies that magnetic field can also produced by changing electric field.